

On NJ-Ternary Rings in Graded Ternary Rings

Hussien Qrewi ^{1*}  Abdullah A. Khatib ²  Hasan Qrewi ³ 

^{1*, 2} Assistant Professor in the Department of Mathematics, Faculty of Science, at Al-Furat University

³ Lecturer, Maaref University of Applied Sciences, Syria

Corresponding Author: e-mail: qrewi@alfuratuniv.edu.sy

Available online at: <https://doi.org/10.66422/RJMU.ID.78>

Received: 2 May 2026

Revised: 15 June 2026

Accepted: 14 July 2026

Abstract— This paper introduces the concept of an NJ-ternary ring and investigates its properties within the framework of graded ternary rings. Particular attention is given to the relationship between a graded NJ-ternary ring and its homogeneous components. It is shown that the identity component of a graded NJ-ternary ring is itself an NJ-ternary ring, whereas every non-identity homogeneous component consists entirely of quasi-regular elements. These results provide further insight into the structural behavior of NJ-ternary rings under grading. The study also examines the relationship between JR-ternary rings and NJ-ternary rings in the graded setting, highlighting connections between these classes of ternary rings and their associated radical properties. The findings contribute to the development of the theory of graded ternary rings and provide a foundation for further investigation of NJ-type structures and Jacobson radical behavior in ternary algebraic systems.

Keywords— Ternary ring, Quasi-element, NJ-ternary ring, Graded ternary ring, Jacobson radical.

Introduction

Algebraic structures are an important branch of mathematics; they have many applications in many fields such as computer science, information science, and coding theory, among others. The theory of algebraic systems was studied by Lehmer [1] in 1932, but earlier such structures were investigated and studied by Prüfer in 1924, Baer in 1929. The notion of ternary rings was introduced by Lister [2]. Graded Ternary rings were introduced by Qrewi [9].

Ternary rings have attracted attention because they extend the usual algebraic structures by using a ternary multiplication. Several concepts have been studied in ternary rings, such as ideals, regular elements, quasi-regular elements, and the Jacobson radical. These concepts help in understanding the structure of ternary rings and the properties of their elements.

Graded ternary rings give another way to study ternary rings by dividing the ring into components according to a grading semigroup or monoid. This makes it possible to study the relation between the properties of the whole ternary ring and the properties of its components. In particular, it is interesting to know

whether a certain property of a ternary ring is also satisfied by some of its graded components.

In this paper, we introduce the notion of an NJ-ternary ring, where every element is either regular or quasi-regular. We study this notion in graded ternary rings and investigate the relation between an NJ-ternary ring and its components. We also consider related concepts such as the Jacobson radical, JR ternary rings, and J-semisimple ternary rings in the graded setting.

2. NJ-Ternary Rings in Graded Ternary Rings:

In this section, we recall the definitions of a ternary ring and a graded ternary ring, along with some properties needed for our study.

Definition 1 (see [2]).

A nonempty set T , equipped with an addition operation and a ternary multiplication, is called a ternary ring if T is an additive commutative group satisfying the following conditions:

- (i) $(abc)de = a(bcd)e = ab(cde)$.
- (ii) $(a + b)cd = acd + bcd$.
- (iii) $a(b + c)d = abd + acd$.

$$(iv) \quad ab(c + d) = abc + abd.$$

For all $a, b, c, d, e \in T$.

Definition 2. (see [7]).

A ternary ring T is called commutative if:

$$abc = acb = bac = bca = cab = cba \quad \forall a, b, c \in T$$

Definition 3 (see [7]).

Let T be a ternary ring. If there exists an element $0 \in T$ such that

$$0 + x = x \text{ and } 0xy = x0y = xy0 = 0 \text{ for all } x, y \in T,$$

then 0 is called the zero element of the ternary ring T . In this case, we say that T is a ternary ring with zero.

Definition 4 (see [7]).

Let T be a ternary ring. If there exists an element $e \in T$ such that

$$eex = exe = xee \text{ for all } x \in T,$$

then e is called an identity element of T . Throughout this paper, T will denote a ternary ring with zero and identity element 1 .

Definition 5 (see [7]).

Let T be a ternary ring. An element $x \in T$ is said to be right (left, lateral) invertible if there exist $y, z \in T$ such that

$$xyz = 1 \text{ (respectively } yzx = 1, yxz = 1)$$

If x is left, right, and lateral invertible, then x is called an invertible element of T . The set of all invertible elements of T is denoted by U_T .

Definition 6 (see [8]).

An additive subgroup A of a ternary ring T is called a ternary subring of T if $abc \in A$ for all $a, b, c \in A$.

Definition 7 (see [8]).

An additive subgroup A of a ternary ring T is called a **left (right, lateral) ideal** of T if:

$$bca \in A \text{ (respectively } abc \in A, bac \in A) \text{ for all } b, c \in T \text{ and } a \in A.$$

A is called an ideal of T if it is a left, right, and lateral ideal of T . An ideal A of T is called a proper ideal if $A \neq T$.

Definition 8 (see [1]).

An element x of a ternary ring T is called right quasi-regular if there exists an element $y \in T$ such that

$$x + y + xty = 0$$

for all $t \in T$. The notion of left quasi-regularity can be defined similarly. An element x of a ternary ring T is called quasi-regular if it is both left and right quasi-regular.

Definition 9 (see [5]).

An element a of a ternary ring T is called regular (in the sense of von Neumann) if there exist two elements $x, y \in T$ such that

$$a = axaya.$$

Definition 10.

A ternary ring T is called an NJ-ternary ring if every element of T is either a regular or a quasi-regular element.

Definition 11 (see [6,8]).

An ideal A of a ternary ring T is said to be a maximal (right, left) ideal if A is a proper (right, left) ideal of T and is not properly contained in any proper (right, left) ideal of T . The sets of all right and left maximal ideals of T are denoted by M_r and M_l , respectively.

Definition 12 (see [6]).

The right and left radicals of a ternary ring T are defined, respectively, by

$$Rad_r(T) = \bigcap_{M \in M_r} M \text{ and } Rad_\ell(T) = \bigcap_{M \in M_\ell} M$$

Lemma 1 (see [6]).

Let T be a ternary ring. Then,

$$Rad_r(T) = \{r \in T; 1 - rst \text{ is a right invertible in } T \forall s, t \in T\}$$

and

$$Rad_\ell(T) = \{r \in T; 1 - str \text{ is a left invertible in } T \forall s, t \in T\}$$

It should be noted that $Rad_r(T) = Rad_\ell(T)$

Definition 13 (see [6]).

We define $Rad(T) = Rad_r(T) = Rad_\ell(T)$

and call it the Jacobson radical of the ternary ring T .

Definition 14 (see [3]).

A ternary ring T is called J-semisimple if

$$Rad(T) = \{0\}.$$

Definition 15.

An element x of a ternary ring T is said to be JR if it is the sum of a regular element and an element of the Jacobson radical. A ternary ring T is said to be JR if all its elements are JR .

Definition 16 (see [5]).

Let T be a ternary ring. If A, B , and C are three subsets of T , we denote the set:

$$ABC = \left\{ \sum abc \quad ; a \in A, b \in B, c \in C \right\}$$

Definition 17 (see [9]).

Let T be a ternary ring and let S be a semigroup. T is called an S -graded ternary ring if there exists a nonempty family $\{T_s\}_{s \in S}$ of additive subgroups of T such that:

$$(i) \quad T = \bigoplus_{s \in S} T_s.$$

$$(ii) \quad T_\alpha T_\beta T_\gamma \subseteq T_{\alpha\beta\gamma} \text{ for all } \alpha, \beta, \gamma \in T.$$

A graded ternary ring-S is denoted by (T, S) . The elements of T_s are called homogeneous elements of degree s . Every $x \in T$ can be written uniquely as

$$x = \sum_{s \in S} x_s ; x_s \in T_s$$

where x_s is the component of x in T_s and $x_s = 0$ for all but finitely many $s \in S$. The support of (T, S) is defined by

$$\text{supp}(T, S) = \{s \in S; T_s \neq \{0\}\}$$

Example :

Lemma 2 (see [9]).

Let S be a monoid with neutral element e , and let $T =$

$\bigoplus_{s \in S} T_s$, T_s be an S -graded ternary ring. Then:

- 1) T_e is a ternary subring of T .
- 2) Suppose that S is multiplicatively cancellative and that $\alpha\beta \neq e$ for all $\alpha, \beta \in S$ such that $(\alpha, \beta) \neq (e, e)$. If

$$1 = \sum_{s \in S} x_s ; x_s \in T_s$$

then,

- for every $h \in S$ and $a_h \in T_h$,

$$a_h = x_e x_e a_h = x_e a_h x_e = a_h x_e x_e$$
- $1 \in T_e$.

$$x = \sum_{s \in S} x_s, y = \sum_{s \in S} y_s ; x_s, y_s \in T_s$$

Hence:

$$\begin{aligned} a &= a \left(\sum_{s \in S} x_s \right) a \left(\sum_{s \in S} y_s \right) a = a \left(x_e + \sum_{s \in S - \{e\}} x_s \right) a \left(y_e + \sum_{s \in S - \{e\}} y_s \right) a \\ &= \left(a x_e a + \sum_{s \in S - \{e\}} a x_s a \right) \left(y_e a + \sum_{s \in S - \{e\}} y_s a \right) \\ &= a x_e a y_e a + \sum_{s \in S - \{e\}} a x_e a y_s a + \sum_{s \in S - \{e\}} a x_s a y_e a + \left(\sum_{s \in S - \{e\}} a x_s a \right) \left(\sum_{s \in S - \{e\}} y_s a \right) \end{aligned}$$

Since:

$$\begin{aligned} (a x_e a) y_s a &\in T_e T_s T_e \subseteq T_s \quad \forall s \in S - \{e\} \\ (a x_s a) y_e a &\in T_s T_e T_e \subseteq T_s \quad \forall s \in S - \{e\} \\ (a x_s a) y_s a &\in T_s T_s T_e \subseteq T_{s^2} \quad \forall s \in S - \{e\} \\ a, a x_e a y_e a &\in T_e \text{ \& } s s \neq e \quad \forall s \in S - \{e\} \end{aligned}$$

Since $T = \bigoplus_{s \in S} T_s$ is a direct sum and $a \in T_e$, the

uniqueness of the homogeneous decomposition implies that all non-identity homogeneous

Standing Assumptions. Throughout the main results, (T) denotes a ternary ring with zero and identity (1) , and (S) denotes a monoid with neutral element (e) . Any additional assumptions on (S) , such as multiplicative cancellativity, will be stated explicitly in the corresponding theorem.

3. Main Results:

Theorem 1.

Let S be a monoid with neutral element e , and let T be a graded ternary ring by S . If T is an NJ-ternary ring, S is multiplicatively cancellative, and $\alpha\beta \neq e$ for every two elements $\alpha, \beta \in S$ such that $(\alpha, \beta) \neq (e, e)$, then:

- 1) The ternary subring T_e is an NJ-ternary ring.
- 2) If $h \neq e$, then every element of T_h is quasi-regular in T .

Proof:

Since $T = \bigoplus_{s \in S} T_s$ is a graded ternary ring graded by monoid S with neutral element e , S multiplication cancellative and $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$, then $1 \in T_e$ (lemma 2).

- 1) Let a be an element of T_e . Since T is NJ-ring, then either a is regular element in T or quasi-regular in T .

- **Case 1: If a is regular element in T** , then there exist in T two elements x, y such that

$$a = axaya. \text{ Since } x, y \in T = \bigoplus_{s \in S} T_s, \text{ then:}$$

components on the right-hand side must vanish.

Hence, $a = ax_e a y_e a$. Therefore a is a regular element in T_e .

- **Case 2: If a is a quasi - regular element in T** , then a is a right and left quasi regular element in T . Since a is a right quasi - regular element in T , then there exist in T

an element b such that $a + b + aab = 0$ for all α of T .

Since $\alpha, b \in T = \bigoplus_{s \in S} T_s$, then:

$$\alpha = \sum_{s \in S} \alpha_s, b = \sum_{s \in S} b_s; \alpha_s, b_s \in T_s$$

Hence:

$$\begin{aligned} & a + \left(\sum_{s \in S} b_s \right) + a \left(\sum_{s \in S} \alpha_s \right) \left(\sum_{s \in S} b_s \right) = 0 \\ \Rightarrow & a + b_e + \left(\sum_{s \in S - \{e\}} b_s \right) + a \left(\alpha_e + \sum_{s \in S - \{e\}} \alpha_s \right) (b_e + \sum_{s \in S - \{e\}} b_s) = 0 \\ \Rightarrow & a + b_e + \left(\sum_{s \in S - \{e\}} b_s \right) + (a\alpha_e + \sum_{s \in S - \{e\}} a\alpha_s) (b_e + \sum_{s \in S - \{e\}} b_s) = 0 \\ \Rightarrow & a + b_e + \left(\sum_{s \in S - \{e\}} b_s \right) + a\alpha_e b_e + \sum_{s \in S - \{e\}} a\alpha_e b_s + \sum_{s \in S - \{e\}} a\alpha_s b_e + \left(\sum_{s \in S - \{e\}} a\alpha_s \right) \left(\sum_{s \in S - \{e\}} b_s \right) = 0 \end{aligned}$$

Since:

$$\begin{aligned} & a, b_e, a\alpha_e b_e \in T_e \text{ \& } a\alpha_e b_s, a\alpha_s b_e \in T_s \quad \forall s \in S - \{e\} \\ & a\alpha_s b_s \in T_{s^2} \text{ \& } ss \neq e \quad \forall s \in S - \{e\} \end{aligned}$$

Since $T = \bigoplus_{s \in S} T_s$ is

a direct sum, the uniqueness of the homogeneous decomposition implies that the T_e -component of the above equality must vanish separately. Therefore,

$$a + b_e + a\alpha_e b_e = 0$$

Hence, a is a right quasi-regular element in T_e

Similarly, the same argument shows that (a) is a left quasi-regular element in (T_e) . Therefore, (a) is a quasi-regular element in (T_e) .

- 3) Since $x, y \in T = \bigoplus_{s \in S} T_s$, then:

$$x = \sum_{s \in S} x_s, y = \sum_{s \in S} y_s; x_s, y_s \in T_s$$

Hence:

$$a = ax_e ay_e a + \sum_{s \in S - \{e\}} ax_e ay_s a + \sum_{s \in S - \{e\}} ax_s ay_e a + \left(\sum_{s \in S - \{e\}} ax_s a \right) \left(\sum_{s \in S - \{e\}} y_s a \right)$$

Moreover ,

$ax_e ay_e a \notin T_s - \{0\}$ because if $ax_e ay_e a \in T_s - \{0\}$, then:

$$\left. \begin{aligned} & ax_e ay_e a \in T_s, (ax_e a)y_e a \in T_{s^3} \quad \forall s \in S - \{e\} \\ & ax_e ay_e a \neq 0, T = \bigoplus_{s \in S} \bigoplus_{s \in S} T_s \end{aligned} \right\} \Rightarrow s = s^3 \Rightarrow se = s^3$$

Since S is multiplicatively cancellative $e = s^2$ and this contradicts $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$.

$ax_e ay_s a \notin T_s - \{0\}$ because if $ax_e ay_s a \in T_s - \{0\}$, then:

$$\left. \begin{aligned} & ax_e ay_s a \in T_s, (ax_e a)y_s a \in T_{s^4} \quad \forall s \in S - \{e\} \\ & ax_e ay_s a \neq 0, T = \bigoplus_{s \in S} \bigoplus_{s \in S} T_s \end{aligned} \right\} \Rightarrow s = s^4 \Rightarrow se = s^4$$

Since S is multiplicatively cancellative $e = s^3$ and this contradicts $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$.

- 2) If $s \in S - \{e\}$ and a is an element of $T_s - \{0\}$, then a is not regular in T because if the element a is regular in T , then there exist in T two elements x, y such that $a = axaya$.

$ax_s ay_e a \notin T_s - \{0\}$ because if $ax_s ay_e a \in T_s - \{0\}$, then:

$$\left. \begin{aligned} ax_s ay_e a \in T_s, (ax_s a)y_e a \in T_{s^4} \forall s \in S - \{e\} \\ ax_s ay_e a \neq 0, T = \bigoplus_{s \in S} \bigoplus_{s \in S} T_s \end{aligned} \right\} \Rightarrow s = s^4 \Rightarrow se = s^4$$

Since S is multiplicatively cancellative $e = s^3$ and this contradicts $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$.

$ax_s ay_s a \notin T_s - \{0\}$ because if $ax_s ay_s a \in T_s - \{0\}$, then:

$$\left. \begin{aligned} ax_s ay_s a \in T_s, (ax_s a)y_s a \in T_{s^5} \forall s \in S - \{e\} \\ ax_s ay_s a \neq 0, T = \bigoplus_{s \in S} \bigoplus_{s \in S} T_s \end{aligned} \right\} \Rightarrow s = s^5 \Rightarrow se = s^5$$

Since S is

multiplication Cancellative, then $e = s^4$ and this contradicts $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$.

Since $ax_e ay_e a, ax_e ay_s a, ax_s ay_e a, ax_s ay_s a \notin T_s - \{0\}$ and $a \in T_s$, then $a = 0$ which is a contradiction.

Since T is an NJ-ternary ring and a_h is not regular in T , it follows that a_h is quasi-regular in T . Therefore, every element of T_h , for $h \neq e$, is quasi-regular in T .

Remark: Theorem 1 shows that the NJ-property descends to the identity component T_e , while all other homogeneous components consist entirely of quasi-regular elements.

Theorem 2.

Let S be a monoid with neutral element e , and let T be an S -graded ternary ring. If T is a JR-ternary ring,

2) that $x = a + b$. Since $a, b \in T = \bigoplus_{s \in S} T_s$, :

$$a = \sum_{s \in S} a_s, b = \sum_{s \in S} b_s; a_s, b_s \in T_s$$

Hence:

$$x = \sum_{s \in S} a_s + \sum_{s \in S} b_s = a_e + \sum_{s \in S - \{e\}} a_s + b_e + \sum_{s \in S - \{e\}} b_s$$

Since:

$$x, a_e, b_e \in T_e \text{ \& } a_s, b_s \in T_s \forall s \in S - \{e\}$$

Then $x = a_e + b_e$. Since a is regular in T , then there exist in T two elements x, y such that $a = axaya$. Since

$$x, y \in T = \bigoplus_{s \in S} T_s, :$$

$$x = \sum_{s \in S} x_s, y = \sum_{s \in S} y_s; x_s, y_s \in T_s$$

Hence:

$$\begin{aligned} a_e + \sum_{s \in S - \{e\}} a_s = \\ (a_e + \sum_{s \in S - \{e\}} a_s)(x_e + \sum_{s \in S - \{e\}} x_s)(a_e + \sum_{s \in S - \{e\}} a_s)(y_e + \sum_{s \in S - \{e\}} y_s)(a_e + \sum_{s \in S - \{e\}} a_s) \end{aligned}$$

Since $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$:

$$a_e = a_e x_e a_e y_e a_e$$

S is multiplicatively cancellative, and $(\alpha\beta \neq e)$ for every $(\alpha, \beta \in S)$ such that $((\alpha, \beta) \neq (e, e))$, then:

1. The ternary subring (T_e) is a JR-ternary ring.
2. If the ternary subring (T_e) is J-semisimple, then (T_e) is an NJ-ternary ring.

Proof:

Since $T = \bigoplus_{s \in S} T_s$ is a graded ternary ring graded by the monoid S with neutral element e , S multiplicatively cancellative and $\alpha\beta \neq e$ for every two elements α, β of S such that $(\alpha, \beta) \neq (e, e)$, then $1 \in T_e$ (lemma 2).

- 1) Let x be an element of T_e , then x is an element of T . Since T is a JR-ternary ring, then there exist a regular element a in T and an element b of the Jacobson radical $Rad(T)$ such

Therefore a_e is regular in T_e . Since $b \in \text{Rad}(T)$,
 $1 - xzb$ is left invertible in $T \quad \forall x, z \in T$ [lemma
1]

So $1 - xzb$ is left invertible in $T \quad \forall x, z \in T_e$.

Therefore $1 - xzb = u$; u is left invertible in
 $T \quad \forall x, z \in T_e$.

Since $u \in T = \bigoplus_{s \in S} T_s$:

$$u = \sum_{s \in S} u_s ; u_s \in T_s$$

Hence:

$$\alpha = \sum_{s \in S} \alpha_s , \beta = \sum_{s \in S} \beta_s ; \alpha_s, \beta_s \in T_s$$

Hence:

$$(\alpha_e + \sum_{s \in S - \{e\}} \alpha_s)(\beta_e + \sum_{s \in S - \{e\}} \beta_s)(u_e + \sum_{s \in S - \{e\}} u_s) = 1$$

Since $1 \in T_e, T = \bigoplus_{s \in S} T_s$, $\alpha\beta \neq e$ for every two
elements

α, β of S such that $(\alpha, \beta) \neq (e, e)$, then $\alpha_e \beta_e u_e =$
1. So u_e is left invertible in T_e .

Hence:

$1 - xzb_e$ is left invertible in $T_e \quad \forall x, z \in T_e$

So $b_e \in \text{Rad}(T_e)$. Since there exist a regular element
 α_e in T_e and an element b_e of the Jacobson radical
 $\text{Rad}(T_e)$ such that $x = \alpha_e + b_e$, then the ternary
subring T_e is JR.

- 3) Let x be an element of T_e . Since the ternary
subring T_e is JR, then there exist a regular
element a in T_e and an element b of the
Jacobson radical $\text{Rad}(T_e)$ such that $x = a +$
 b . Since T_e is J-semisimple, then $\text{Rad}(T_e) =$
 $\{0\}$. Therefore $b = 0$. Hence $x = a$ is regular
element in T_e . Therefore, T_e is an NJ-ternary
ring.

4. Conclusions:

In this paper, we mainly studied NJ-ternary rings in the
context of graded ternary rings. We investigated the
relationship between a graded NJ-ternary ring and its
homogeneous components and established several
results concerning the behavior of regular and quasi-
regular elements in these components. In particular,
under suitable conditions on the grading monoid, we
showed that the neutral component (T_e) inherits the
NJ-ternary ring property from the graded ternary ring
(T). We also examined the quasi-regularity of
elements belonging to the other homogeneous
components (T_s) , where $(s \neq e)$. These results
provide a connection between the structure of an NJ-

$$1 - xz(b_e + \sum_{s \in S - \{e\}} b_s) = u_e + \sum_{s \in S - \{e\}} u_s$$

Therefore:

$$1 - xzb_e - \sum_{s \in S - \{e\}} xzb_s = u_e + \sum_{s \in S - \{e\}} u_s$$

Since $T = \bigoplus_{s \in S} T_s$:

$$1 - xzb_e = u_e \quad \forall x, z \in T_e$$

Since u is left invertible in T , then there exist two
elements α, β in T , such that $\alpha\beta u = 1$. Since

$$\alpha, \beta \in T = \bigoplus_{s \in S} T_s :$$

ternary ring and the algebraic properties of its grading
components.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest.

Funding

The authors received no financial support for this book
review.

Human Participants

This paper does not involve human participants.

Originality Note

The authors confirm that the manuscript is their
original work, and if others' works are used, they are
properly cited/quoted.

REFERENCES

1. D. H. Lehmer, "A ternary analogue of abelian groups," Amer. J. Math., vol. 59, pp. 329–338, 1932.
2. W. G. Lister, "Ternary rings," Trans. Amer. Math. Soc., vol. 154, pp. 37–55, 1971.
3. T. K. Dutta and S. Kar, "On the Jacobson radical of a ternary semiring," Southeast Asian Bulletin of Mathematics, vol. 28, pp. 1–13, 2004.
4. V. R. Daddi and Y. S. Pawar, "Ideal theory in commutative ternary A-semirings," International Mathematical Forum, vol. 7, pp. 2085–2091, 2012.
5. D. Madhusudana Rao and G. Srinivasa Rao, "Special elements of a ternary semiring," Demonstratio Mathematica, vol. 47, pp. 123–130, 2014.
6. M. K. Ahmad and A. H. Alwan, "Local ternary

- rings and cleanness,” Research Journal of Aleppo University, vol. 119, 2017.
7. R. Alhaj Esmaeel Arabi, M. K. Ahmad, and S. Aouira, “A study of hereditary ternary rings,” Research Journal of Aleppo University, vol. 138, 2020.
 8. M. Alabdullah, B. Al-Hasanat, and J. Jaraden, “Noetherian and Artinian ternary rings,” International Journal of Open Problems in Computer Science and Mathematics, vol. 14, no. 1, 2021.
 9. H. A. Qrewi, “A study on graded ternary rings by monoid,” Research Journal of Aleppo University, vol. 167, 2023.

About the Authors:

Hasan Qrewi holds a Ph.D. in Applied Mathematics from Andhra University, India. He is currently a lecturer at Maaref University of Applied Sciences. He has extensive experience in teaching mathematics and academic research and has published several scientific papers, most of which are indexed in Scopus. His research interests include applied mathematics, mathematical modeling, and related interdisciplinary applications.

Abdullah Ahmad Khatib received his Ph.D. in Mathematics from the University of Aleppo in 2024, specializing in Non-Classical Logic Algebra. He has

extensive experience in teaching mathematics at both secondary and university levels. His research interests include classical and non-classical logic algebras, abstract algebra, particularly groups, rings, modules, and lattices, as well as applied mathematics and mathematical programming using MATLAB and C++.

Hussien Qrewi is an Assistant Professor in the Department of Mathematics, Faculty of Science, at Al-Furat University. He received his Ph.D. in Mathematics from the University of Aleppo. He has published numerous research papers and has supervised several master's and Ph.D. theses in algebra. His research interests include algebra, particularly graded rings, graded semirings, and graded ternary rings.



This is an open access article distributed under the terms of the [Creative Commons Attribution-NonCommercial 4.0 International](https://creativecommons.org/licenses/by-nc/4.0/) (CC BY-NC 4.0) which allows reusers to distribute, remix, adapt, and build upon the material in any medium or format for noncommercial purposes only, and only so long as attribution is given to the